THE SECONDARY MAGNETIC FIELD OF THE EARTH.[1]

By A. W. Rücker, Sec. R. S.

The magnetic field of the earth may be regarded as that primarily due to a uniformly magnetized globe disturbed by some secondary causes. The probability that the primary field is produced by a deep-seated system of electrical currents (whose genesis we can not explain), is increased by the fact that iron, magnetite, and basalt all become non-magnetic if the temperature be raised to the neighborhood of a red heat. The same magnetic effects could be produced on the surface either by magnetized matter or by electrical currents properly distributed within the globe. But if the enormous pressures to which the internal portions of the globe are subjected do not affect the properties with which we are dealing, magnetizable matter can only exist very near to the surface. As we penetrate from the surface downwards, the temperature rises by about 1° C. for every 90 feet, or (say) 37° per kilometer, and if this rate of increase is maintained, the temperature at which iron becomes non-magnetic (which is different for different specimens) would be reached at a depth of about 20 kilometers. Some experiments made in my laboratory by Messrs. Barton and Williams indicate that for magnetite the limiting temperature is less (about 557°), and would be reached at a depth of about 15 kilometers. It is convenient to call that stratum in the earth at which all matter ceases to be magnetic, the *magnetic floor*.

Mr. Henry Wilde has imitated the primary field of the earth by an arrangement of currents. Inside a globe eighteen inches in diameter a smaller sphere was placed, and a wire was coiled round this in planes inclined at an angle of 23° 30′ to the plane of the equator. Between the two globes was a spherical shell of wire gauze, round which another wire was coiled in planes perpendicular to the geographical axis. Currents which could be separately regulated could be passed through the two circuits. The whole arrangement was so adjusted that any part of the globe could be brought under a support on which a small compass or dip-needle could be placed at pleasure. The field due to the currents was

[1] The following article is an expanded account of part of a lecture on "Earth Currents and Electric Traction," delivered by Professor Rücker before the Royal Institution, on April 14, 1899.

about ten times the earth's field at the point where the readings were made. The axis of the inner sphere could be made to revolve when inclined at a constant angle to that of the external globe, and an attempt was made to imitate the secular change by this revolution. This attempt was only partially successful, and, for the moment, I propose to discuss only the particular arrangement by which the existing state of the earth's surface was represented.

For any one position of the axis of the internal globe, the whole of the interior arrangements above described are equivalent to a uniform magnetization, parallel to an axis cutting the surface somewhere between the geographical pole and the arctic circle. Such an arrangement, as we know, will not represent the magnetic state of the earth; but after various trials, the history of which I need not recount, Mr. Wilde hit upon the expedient of covering with thin sheet-iron those portions of the interior surface of the outer globe which correspond to the oceans.

The result has been described by himself as follows: "The devious lines of the declination which had hitherto resisted all attempts to reduce them to order, and masked the simplicity of the primary phenomena of terrestrial magnetism set forth in the preceding propositions now presented themselves as secondary phenomena—the effect of the unequal curvature of the terrestrial surface during the secular refrigeration.

"The declination at the Cape of Good Hope, latitude 34° S., which was only 19° maximum without the iron covering, was now 30° W., the amount required for the epoch; while the declination at Cape Farewell was now 42° W.

"The southern hemisphere of the globe also contained two lines of no declination, nearly coincident with those on the charts for the epoch 1880, and four lines of no declination similarly coincident in the northern hemisphere; two of which lines on the North American and European continents being continuations of those in the southern hemisphere. But the most remarkable and unexpected feature of the distribution of the magnetism on the iron covered globe was the reproduction of the oval area of small westerly declination in Eastern Asia, between longitude 110° E., and 160° E.; surrounded by large areas of eastern declination. The oval also agreed in detail with that on the chart in having the largest westerly declination, about 8° in the center, between the lines of no declination.

"Scarcely less interesting was the reproduction of the oval area of small easterly declination about 5°, surrounded by a large area of greater eastern declination in the equatorial parts of the Pacific (120°–170° W.), while the unsymmetrical form of the magnet c equator was very similar in its deviations to that of the terrestrial globe for the epoch 1880." [1]

Mr. Wilde has been kind enough to present one of his globes to the National Collection of Scientific Apparatus at South Kensington. His observations on the agonic lines and the magnetic equator have been repeated, and their general accuracy confirmed by Mr. Forsyth in the laboratories of the Royal College of Science.

The above is, I think, a fair account of Mr. Wilde's "magnetarium" as described by himself. In a later document he refers to the fact that, in addition to the iron used in covering the seas, bands of sheet-iron were placed on "the areas of the mapped globe occupied by the southern mountain ranges of the Asiatic Continent. A similar polarizing band was also required over the South American Continent to bring into adjustment the zero line with that of the chart of the declination."

Mr. Wilde has recently informed me that some other polarizing bands were also used, which apparently were not mentioned in his accounts of his model. It is, therefore, a matter of interest to know how far the close agreement shown in the above map between the real and artificial agonic lines is due to the action of these bands. On this point Mr. Wilde is unable to give me precise information, and as I do not feel justified in dissecting a model deposited at South Kensington, it will be desirable to investigate the matter anew.

Again, Mr. Wilde only shows the agreement between certain principal lines (agonic, magnetic equator, Pacific oval) as given on the magnetic maps for 1880, and as deduced from his model. Some preliminary experiments lead me to doubt whether the agreement is satisfactory for the intermediate isogonal lines. Thus the two isogonals of 20° westerly declination, which, in 1880, intersected in mid-Atlantic, appear to be much distorted. It must therefore be clearly understood that I think that both the nature of the results attained by Mr. Wilde and the method of obtaining them require further investigation. He has certainly succeeded in imitating the principal magnetic lines by a combination of inducing and induced magnetization, and he attributes his success primarily to the fact that he covered the oceans with thin sheets of iron. It is very desirable,

[1] Quoted from a description of Mr. Wilde's magnetarium, circulated by himself.

before further experiments, which are likely to be tedious and costly, are undertaken, to consider, on *a priori* grounds, whether it is worth while to devote time and attention to a theory which assumes that the earth beneath the oceans is more highly magnetizable than that beneath the continents. In connection with this point it will be necessary to examine in greater detail the conditions under which Mr. Wilde's experiments were performed.

Before investigating the matter further, I must admit that I feel a certain repulsion in dealing with a question of so highly speculative a nature, and I have no doubt that many of my readers will be inclined to put aside the suggestion that the earth beneath the seas is more magnetic than elsewhere as almost unworthy of discussion. On the other hand, it must be remembered that in the past we have had hardly a glimpse of an explanation of the causes of the phenomena of terrestrial magnetism, and that in approaching so difficult a subject we must beware of not breaking the first canon of scientific research, viz., that in studying a phenomenon of which the causes are unknown, we must clear our minds of all *a priori* theories, or at all events use them only as working hypotheses. In spite, therefore, of the fact that we must deal with suggestions which we can never hope to verify by direct experiments; that we must make assumptions as to the constitution of the earth at depths we can not reach,—in spite of the essentially speculative character of the whole inquiry, I think that, with Mr. Wilde's model before us, it is our duty to ask whether there is anything physically absurd and scientifically impossible in the hypotheses to which the construction of that model points.

Apart, then, from the crude statement that from some unknown cause magnetic matter has accumulated under the seas, is there any reason for supposing that, if there is magnetizable matter within the earth, there would probably be a thicker layer of it beneath the oceans than beneath the continents? To this we can at once answer that unquestionably there is, and that it was indicated by Mr. Wilde himself. The *Challenger* expedition proved that the bottom of the deep ocean is at a temperature only a few degrees above that of melting ice. The surface temperature, of course, varies with the latitude; but the bottom of the deep sea is everywhere very cold, and even in cases like that of the Mediterranean, where the communication with the Arctic regions is practically barred by the comparatively shallow straits of Gibraltar, the bottom of the sea is at a temperature far below that attained at an equal depth below the surface of the land.

The average depth of the ocean is about 4 kilometers, and thus, if we assume that we are dealing with rocks of uniform thermal conductivity and that the bottom of the ocean and the surface of the land are of the same temperature (a most unfavorable assumption in the case of the tropics), the magnetic floor, or the isothermal surface corresponding to the temperature of no magnetization, would be about 4 kilometers deeper within the surface of the globe under an ocean of average depth than under a continent. Beneath the deepest parts of the ocean the difference in the position of the magnetic floor would be increased.

If, then, we suppose that at and for some distance below a depth of 20 kilometers the earth is composed of matter which is magnetic below a certain temperature, and is of the same thermal conductivity as rock, no part of this material would be magnetizable under the continents, while under the oceans a layer of an average thickness of 4 kilometers would be magnetized by the earth's field.

If the surface of the shell of magnetizable matter is at a less depth than 20 kilometers, it would be magnetized everywhere; but the magnetized portion would, on the average, be 4 kilometers thicker beneath the ocean than elsewhere.

If the surface of the shell is at a depth greater than 20 km., but less than that distance plus the greatest ocean depths—say less than 28 km.—the average thickness of the magnetized portion beneath the sea would be less than 4 km.

It thus appears that if the earth is composed of magnetizable materials, or if there is in the earth a thick shell of magnetizable matter of which the depth of the upper surface does not exceed 20 km., and if the great pressures to which such materials would be subjected do not completely modify their magnetic properties, the thickness of the layer below the temperature of no magnetization will be greater beneath the seas than elsewhere. Hence this additional thickness may be the physical fact which Mr. Wilde has represented by thin sheets of iron covering the ocean areas.

Since we have, then, not only a very remarkable agreement between the model and the magnetic state of the surface of the earth, but also a *vera causa* which indicates that the model may be an approximate representation of the true physical state of the earth, it remains to discuss whether the disturbances of the earth's magnetic field could really be produced by magnetic matter of which the upper surface is at a depth not exceeding 20 km. and of thickness comparable with 4 km.

For in several important points Mr. Wilde's model differs from the true state of the earth, and in some respects the experiments are imperfect. Thus, (1) the magnetic forces at different points have not been compared; (2) the central points of the compass and dip-needle employed are not at the same distances from the surface of the globe. If that surface is to be taken as corresponding to the surface of the earth, the centers of the declination and dip-needle were at distances from it which on the natural scale would correspond to 700 and 1,400 km. respectively. Taking the mean of these as about 1,000 kilometers, it is evident that the observations were made at a distance at which the form of the magnetic field would be appreciably different from that close to the surface of the globe. In the model, relatively powerful forces would be produced close to the edges of the iron plates, to which nothing on the surface of the earth corresponds. These might be much reduced by supposing the plates to thin out gradually, or to diminish in permeability, as the edges are approached. But apart from a point of this kind, in which the model could not be expected to imitate nature exactly, the more prominent features of the earth's field would be modified at a distance from the earth's surface of about one-sixth of its radius to an extent which could be calculated from the Gaussian expansion. Thus it was found to be important to cut a hole in the iron coating roughly corresponding to the West Indian Islands. The length of this is comparable with the distance of the center of the declination needle from the earth's surface; the breadth must be much less. It is evident that the effects of such a gap in the magnetic field would be very different close to it and at a distance from it comparable with its largest dimension; and, further, that the effects as measured by needles about 700 kilometers in length would be very different from those actually recorded in a magnetic survey.

All these arguments, therefore, tend to prove that if the magnetic matter represented by Mr. Wilde's iron sheets has a real existence, it must be at a depth of 1,000 kilometers below the surface of the earth, and that in his experiments the radius of the earth must be taken to be the mean distance of the centers of his exploring magnets from the center of the globe. Unless, therefore, these arguments can be met, the hypothetical magnetic matter will be beneath the magnetic floor, and its existence could only be accounted for by an unwarranted assumption as to the effects of pressure on the magnetic properties of bodies.

An attempt might be made to evade the difficulty by assuming that the effects represented by the magnetization of the iron coating are really due to systems of electric currents, but no good reason can be assigned why these should closely simulate the presence of a magnetic shield.

On the whole, then, we can not adopt a more favorable view than that if it were shown to be possible to reproduce as good a representation of the magnetic state of the earth's surface as Mr. Wilde has made, when the exploring needles were close to the iron coating, and when the gaps and edges were replaced by more widespread and gradual changes of permeability, it might be possible to regard the experiment as having been performed under conditions consistent with the view that the shielding material is above the magnetic floor.

It is, however, worth while to assume that these difficulties are overcome in order to inquire whether, even in that event, any insuperable physical obstacle would be revealed. It might, for instance, be capable of proof that the irregularities in the magnetic field of the earth could only be produced by an iron shell much thicker than the distance between the surface of the earth and the magnetic floor. Such points must, therefore, be considered before any further labor is expended on the investigation.

The thickness of the iron plates used by Mr. Wilde was 0·012 inch = 0·48 mm., and as they were "tinned" the actual thickness of the iron would be somewhat less than this. Now, as the radius of the globe was 9 inches, 0·012 inch = 5·2 miles or 8.4 kilometers, which is greater than the average depth of the ocean, but less than the depth of the magnetic floor below the bottom of the ocean.

As far, therefore, as the plate used is concerned, no physical impossibility is revealed; but this argument proves very little, for the material of the magnetic layer which the iron coating represents, the magnetic field in which it is placed, and its temperature, would all be different from those of iron coating itself.

It may be useful, therefore, to consider the matter more fully.

Dr. Bauer has recently published a map of the residual field of the earth, obtained when the forces due to a uniform magnetization are subtracted from those which are actually observed on the surface. If we assume that this residual field is due to Mr. Wilde's magnetic layer, we may get some idea of what the thickness of that layer must be for a given permeability. At the points where the residual field is most powerful the forces amount to about ± 0·4 of the calculated uniform field. This ratio is attained at a few points only. The average ratio of the residual to the uniform field would be about $\pm$ 0·1 or $\pm$ 0·2 of the latter.

Now, if we suppose the earth to be surrounded by a uniform magnetic shell shielding the force produced by an internal uniformly magnetized sphere, non-magnetic, it is evident that, by cutting holes in the shell, or making parts of it non-magnetic,[1] we could impose upon the primary field secondary disturbances, the ratio of which to the unshielded field would depend upon the form of the holes and the thickness and permeability of the shell.

The whole potential would consist of two parts,—that due to the internal inducing sphere, and that due to the magnetism induced in the fragments of the shell. If the latter part, when expanded in spherical harmonics, contains a term corresponding to uniform magnetization, then the corresponding term in the expansion of the whole potential outside the shell will not give the potential due to the uniform magnetization of the internal sphere, but that potential modified by the term due to the magnetization of the shell.

If, then, we suppose Dr. Bauer's argument to be applied to a globe constituted as Mr. Wilde's model assumes, the primary magnetization, as calculated, may differ appreciably from that really due to the internal sphere, and thus an appreciable error may be introduced into the determination of the secondary field. Of course Dr. Bauer was well aware of this fact, and his method is perfectly legitimate as an approximation; but when we compare the results with a physical theory, it is important to remember that the true primary and secondary fields, defined with reference to inducing and induced systems of magnetization, may differ considerably from those defined with reference to the first and following terms of a Gaussian expansion.

While on this point I may also remark that the interesting suggestion made by Dr. Bauer, in the last number of *Terrestrial Magnetism*, on a possible connection between the secondary magnetic field of the earth and the isabnormals of temperature, is open to the objection that the deviation of the average temperature of any points at the surface of the ocean from the average for the corresponding latitude is a surface phenomenon, which at points near the bottom of the ocean is completely masked by the influx of cold water.

[1] It is hardly necessary to point out that, though the magnetic effects of these two arrangements would be identical, the one would and the other would not produce gravitational irregularities on the surface.

His theory would therefore compel us to look for the source of the secondary magnetic field at depths less than the ocean bottom.

Taking Dr. Bauer's numbers as they stand, I have determined the ratio of the secondary or residual to the primary forces at a few points, and, as has been stated, the results indicate that, on the average, the resultant is about 1·2 of the primary force in regions where the secondary force strengthens it, and about 0·8 where the two forces are opposed. These figures do not pretend to any great accuracy, but are sufficiently near the truth for preliminary calculations.

Now, I take it that if we calculate the thickness of a uniform shell of given permeability, which would reduce the intensity of the true primary field in ratios similar to those given above, we shall get an idea of the order of the magnitude of the thickness of the broken shell necessary to produce the observed variations. The method is very rough, and would be of very little use in extreme cases; but it will serve to indicate whether there is any physical impossibility in Mr. Wilde's assumptions. We will take two ratios of the shielded to the unshielded field; viz., the ratio of the smaller secondary to the primary field—*i. e.*, 0·8—and the ratio of the smaller to the larger secondary field—*i. e.*, 0·8 / 1·2 = 2 / 3. The argument will be strengthened if somewhat similar results are obtained when we substitute the maximum for the average secondary forces. In this case the two shielding ratios corresponding to the above would be 0·6 and 0·6 / 1·4 = 3 / 7.

If, then, t be the thickness of a shell of large permeability μ, and if a be the radius of the shell which is large compared with t, the ratio of the shielded to the unshielded field, E, in the case under consideration, is given by the formula:

$$E = \frac{3}{3 + 2t\mu / a}$$

Hence, substituting the above values for E, we get the following results:

TABLE I

E	=3/5	3/7	4/5	2/3
$\frac{t\ \mu}{a}$	1	2	3/8	3/4

The second of these values of $t\mu/a$ is almost certainly too large. It is based upon the assumption that in order to produce, by means of a pierced shell, variations in the forces between a certain maximum (M) and minimum (m) value of the ratio of the disturbed force at any place to the force at that place derived from the first term in a Gaussian expansion, it is necessary to have a shell which, if complete, would reduce the forces produced by an internal uniformly magnetized sphere in the ratio M/m. This is manifestly exaggerated if we take edge effects into account, and is, I think, too large, even if we omit regions very near to the edges.

We next have to decide what value we may assume for the permeability. The three materials which we may regard as being, with greater or less probability, at our disposal, are virgin iron, magnetite, and basalt. The permeability of very magnetic specimens of the last material is only of the order 1·01, and it may be put aside as quite incompetent to produce the observed effects.

As far as I am aware, the most trustworthy measurement hitherto made of the permeability of magnetite is an unpublished experiment of Professor S. P. Thompson's, which he kindly allows me to quote. Using the ring method, he found the permeability in small fields to be about 4. Both Messrs. Barton & Williams (loc. cit.) and I (Proc. Roy. Soc. 48, 1890, p. 592) have shown that the permeability of magnetite rises with temperature; but on the basis of Professor Thompson's experiment, I do not think that we can assume that it would rise as high as 10. On the other hand, different specimens of a natural mineral, such as lodestone, may differ enormously in permeability,[1] and it would be unsafe to base any definite conclusions on a single experiment. All that can be said is, that unless future observations should show that the permeability of large masses of natural magnetite is of the same order as that of iron, it can not be the shielding substance.

Taking the largest of the ratios of the shielded to the unshielded fields given in Table I, viz. 4/5, the thickness of shell of permeability 10 which would produce this reduction would be about 5 per cent of the radius. Hence, in the case of the earth, the shell of magnetite would be at least 315 kilometers in thickness. As this is fifteen times the depth of the magnetic floor, it is evident that Mr. Wilde's iron shell can not represent a shell of magnetite, unless further investigation shows that the average permeability of that material is

[1] Another experiment of Professor Thompson's, but not made by the ring method, indicates that this may be the case.

far above that of Prof. S. P. Thompson's specimen, or unless the effects of great pressure neutralize those of temperature.

Considering next the case of iron itself, we know (1) that the permeability of impure specimens may be very low, and (2) that the permeability at ordinary temperatures is small in small fields. On the other hand the late Dr. Hopkinson has shown that, in fields comparable with that of the earth and at a high temperature, the permeability of wrought iron may amount to 11,000. On the whole, I do not think it absurd to suppose that the permeability of the iron shell (if it exists) may be taken as 1,000.

The meteoric theory of the constitution of the earth suggests the possibility of a shell of meteoric iron; and as this substance contains nickel, and some alloys of nickel and iron are very slightly magnetic, this possibility may be regarded as weakening the case for the existence of the shell.

I can not find any record of a measurement of the permeability of meteoric iron, and though I hope to supply this want, it will be dangerous to argue from results obtained with a single specimen.

There are, however, a number of large iron meteorites in the Natural History Department of the British Museum, and by the courtesy of my friend, Mr. Fletcher, F. R. S., the keeper of the minerals, I have been allowed to test their magnetic condition. I found that they were all strongly polarized by induction as though by the earth's field, and on turning some of the smaller specimens upside down the direction of magnetization was reversed.

The largest of these specimens was the famous Cranbourne meteorite discovered near Melbourne, in Australia, which was examined *in situ* by Dr. Neumayer in 1861. He found it to be strongly magnetized as though by induction. It is magnetized as though by induction in its present position. All the evidence, therefore, points to the conclusion that meteoric iron is fairly soft and permeable.

Taking then the radius of the earth as 6,400 kilometers, and μ as 1,000, we get the following shell thicknesses in kilometers from Table I:

TABLE II

$\frac{t\,\mu}{a}$	1	2	⅜	¾
t	6·4	12·8	2·4	4·8

Now, remembering that the second of these is almost certainly too large, we are justified in saying that the shell-thicknesses calculated on several hypotheses (all of which are very rough) are of the order of the mean depth of the ocean (4 km.), and that this purely provisional and preliminary argument, on the whole, supports the view that if the magnetic material required by Mr. Wilde has a permeability comparable with 1,000, the effects he imitates could be produced by a shell whose thickness does not differ very much from the average depth of the ocean. Pending further measures on the permeability of magnetite, it appears that iron is the only known substance which will satisfy the requirements of the case.

EFFECTS OF THERMAL CONDUCTIVITY.

Up to the present, I have assumed that the thermal conductivities of the materials employed are but little different. Our arguments, with this limitation, have pointed to the conclusion that if the magnetic matter indicated by Mr. Wilde's model really exists, and if the magnetic properties of bodies are approximately the same under great and small pressures, the hypothetical magnetic shell must itself be of iron. As the thermal conductivity of iron is about thirty times greater than that of stone, a very large addition may be made to the possible thicknesses as above calculated.

The gradient of temperature in the iron shell will be thirty times less than in the rock, and whereas the two faces of a stone shell 4 km. in thickness would differ by 148° C., the difference of temperature if the shell were made of iron would be only 4·9° C. This advantage might be to a certain extent neutralized by the fact that the better conductivity of the iron would tend to equalize the temperature of all parts of the shell; but though this would smooth off the effects of relatively small elevations or depressions, it can easily be shown that it would not affect large areas.

Taking a special case, consider an infinite solid, bounded by a plane surface towards which there is a gradient of temperature of 37° C. per kilometer. At a depth of 4 km. the temperature would be 148° C. Superpose, on this system, at the depth of 4 km. a variation of temperature which, measured parallel to a given line (X) in the surface, is given by the formula:

$$74 \left\{ \sin \frac{\pi x}{X} - 1 \right\} \quad .$$

Hence, there will be alternate maxima of 148° C. (the original temperature of the layer) and minima of 0° C., the transition being very gradual, in accordance with the simple harmonic law.

We can make the change as sudden as we please by using (with an appropriate constant in the place of $\frac{\pi}{4}$) more and more terms of the formula:

$$\frac{74\times 4}{\pi}\left\{-\frac{\pi}{4}+\sin\frac{\pi x}{X}+\tfrac{1}{3}\frac{\sin 3\pi x}{X}+\tfrac{1}{5}\frac{\sin 5\pi x}{X}+\ldots\right\}\quad .$$

Taken to infinity, the Fourier series is $\frac{\pi}{4}$ between $x=o$ and $x=\pi$ and $-\frac{\pi}{4}$ between $x=\pi$ and $x=2\pi$; *i. e.*, it represents alternate hot and cold regions of breadth X, with discontinuities between them.

It will be sufficient for our present purpose to take the simpler form, which includes only the first term. We shall therefore have a distribution of temperature in the solid given by the formula:

$$V=148+37y+74\left\{\left(Ae^{\frac{\pi y}{X}}+Be^{\frac{-\pi y}{X}}\right)\sin\frac{\pi x}{X}-1\right\}$$

Where y is measured in km. positive downwards from the place 4 km. below the surface, and A and B are arbitrary constants subject to the condition $A+B=1$. In the case considered, $A=0$ and $B=1$.

Next, replace the portion of the matter contained between y_1 and y_2 by a material the thermal conductivity of which is thirtyfold greater than that of the rest of the solid. As y is measured from a plane which is itself 4 km. below the surface, the whole of the new material is below the magnetic floor, as previously defined.

It is easy by well-known methods to find the temperature under the new conditions in all parts of the compound solid.

It is sufficient to consider the lower surface of the plate, which we will suppose to be at a depth y_2 from the bottom of the mean ocean. If the whole solid were of uniform material the potential at this depth due to the harmonic distribution of temperature would be:

$$74e^{\frac{-\pi y_2}{X}}\sin\frac{\pi x}{X}$$

After the introduction of the plate, it is

$$74\, B_2\, e^{\frac{-\pi y_2}{X}} \sin \frac{\pi x}{X}$$

where

$$B_2 = \frac{4\, k}{(\xi_2^{-2} - \xi_1^{-2})\,(k^2 - 1) + (k+1)^2 - \xi_1^2\, \xi_2^{-2}\,(k-1)^2}$$

and if y_1 is the depth of the upper surface of the plane,

$\xi_1 = e^{\frac{\pi y_1}{X}}$, $\xi_2 = e^{\frac{\pi y_2}{X}}$ and k is the ratio of the conductivity of the plate to that of the remainder of the material.

Hence, the difference between the maximum and minimum temperatures at this depth when the solid is uniform, or is disturbed by the introduction of the plate is:

$$2 \times 74\, \xi_2^{-1}\,(1 - B_2)$$

Now it is evident that this difference of range is affected oppositely by two causes; firstly, the thermal resistance between the surface of the solid and the lower surface of the plate is diminished by the introduction of the plate, which will tend to increase the range of temperature at the lower surface; secondly, the exchange of heat between different parts of the plate is facilitated, which will tend to reduce that range.

The first effect will be more prominent if the upper surface of a thin plate is very near the surface of the solid; the second will be predominant if the plate is at such a depth that its thickness does not materially facilitate the passage of heat to the lower plane.

Now, $B_2 = 1$ when $\xi_1 = \xi_2$, or $\xi_1^2 = (k+1)\,(k-1)$. It is only, therefore, when the upper surface of the plate is at a depth greater than that given by this limit that the range of temperature at the lower surface will be adversely affected by the lateral conductivity of the plate.

In the case under consideration, $k = 30$, and if we take $X = 3{,}600$ km., or about 2,200 miles, we shall have alternate strips of "continent" and "ocean" of that breadth. Hence, $B_2 = 1$ when

$$e^{2\pi y_1/3{,}600} = 31/29;\ \textit{i. e.}, \text{ when } y_1 = 38 \text{ km.}$$

Hence, as this depth is greater than that at which we should place the upper surface of the plate, we find that its presence increases the range of temperature at its lower surface. For other terms of the Fourier series the limiting value of y_1 would be diminished in proportion to the exponents of *e*; *i. e.*, for the three next terms it would be 12.6, 7.6, and 5.6 km. respectively.

As these depths are less than that at which the upper surface of the plate need be placed, the indirect conductivity effect of these terms will tend to equalize the temperatures of different parts of the plate; but they also tend to increase the average difference between the sub-continental and sub-oceanic regions.

It is hardly necessary to elaborate the point further; but if we take $y_1=18$ and $y_2=27$ km., the depths of the boundaries of the plate below the surface of the globe are 22 and 31 km. respectively. Their temperatures will be 814° C. and 1,147° C., if the material were rock; and 814° C. and 825° C. only, if the plate were iron. Superpose at the depth of 4 km. the variation given by the formula

$$74 \left\{ \sin \frac{\pi x}{3{,}600} - 1 \right\} .$$

This, added to the temperatures due to the vertical flow of heat, makes the maximum temperature under the continent 148° C., and the minimum under the sea 0° C. If the material were uniform, the temperature variation at 27 km. would be:

$$74\, e^{-27\pi/3{,}600} \sin \frac{\pi x}{3{,}600} = 72 \sin \frac{\pi x}{3{,}600} .$$

The introduction of the iron plate does not modify this to an important extent, and thus the temperature of the lower surface as given by the formula:

$$825+72 \left(\sin \frac{\pi x}{3{,}600} - 1 \right)$$

giving, for mid-continent, shore line, and mid-ocean, the values 825°, 753°, and 681° C., respectively. At the upper surface, these would all be diminished by about 11° C.

If we take the 742° C. as the temperature of non-magnetization, it will cut the lower plane at the point where

$$\sin \frac{\pi x}{3{,}600} = \frac{11}{72} = 0.15 .$$

Hence, $x=170$ km., or, say, 110 miles. To this distance must be added that due to the fact that the permeability will not reach its maximum value until a certain temperature above 742° is attained, so that perhaps a range of 200 or 300 miles in which the magnetic part of the shell thickens and increases in permeability, and by which edge effects will be smoothed off.

On the whole, then, even if the approximate thicknesses calculated from the consideration of the shielding of complete shells are too small, the additional range of thickness conferred by the conductivity of the iron, would probably do away with all difficulties as to the possibility of introducing a shell of the requisite shielding efficiency above the magnetic floor.

LAMINATION.

One other point may be noticed. Under certain circumstances the shielding effect of a spherical shell is improved if it is divided into two concentric shells separated by a spherical crack, and injured if the gap is filled up with magnetic material. (In the lecture this was illustrated by coaxial cylinders filled with iron filings.) Thus, if a_1 and a_0 are the external and internal radii of a spherical shell, lamination will be injurious if

$$\frac{a_1^3}{a_0^3} < 1 + \psi$$

where ψ is a function of the permeability μ of the form

$$\psi = \frac{9\mu}{(2\mu + 1)(\mu + 2)}$$
$$= 9/2\mu, \text{ if } \mu \text{ is large.}$$

If the external radius and the amount of shielding material are given, it may be better to divide the shell into two shells before the thicknesses given by the above formula are reached, but a better result can always be obtained by filling up the gap with the magnetic matter until the limit given by the equation,

$$\frac{a_1^3}{a_0^3} = (1 + \sqrt{\psi})^2 \text{ is attained.}$$

For thicker shells than those whose inner radius is given by this formula it is positively injurious to fill the whole of the shielding space with the permeable material.

Below this limit, lamination must diminish the shielding.

$$\text{Now if } \mu = 1{,}000 \quad \psi = 0{\cdot}0045$$
$$\therefore (1 + \psi)^{1/3} = 1{\cdot}0015$$
$$\& \ (1 + \sqrt{\psi})^{2/3} = 1{\cdot}045$$

Hence, the advantage of lamination, when the quantity of material is restricted, would begin, in the case of the earth, when the thickness was about 10 kilometers; and, if the quantity is not restricted, when the thickness is about 270 kilometers.

These results would not hold good for a pierced shell; but they are sufficient to show that, with thicknesses such as those to which we should probably be restricted in an accurate solution of the problem, no great advantage is likely to be gained by the use of the principle of lamination.

SUMMARY.

The result of the foregoing discussion may be briefly summarized as follows:

Mr. Wilde has produced a good magnetic model of the globe by means of an arrangement which consists in effect of a primary field due to a uniformly magnetized sphere, and a secondary field due to iron placed near the surface and magnetized by induction. The principal part of the iron is placed under the oceans; but evidence is not forthcoming as to how far the close agreement between the natural and artificial agonic lines and magnetic equators is due to certain "polarizing strips" which were placed under the land. Mr. Wilde attaches the greatest importance to the covering of the oceans with iron.

As the result of a purely preliminary discussion it appears that unless great pressure profoundly modifies the properties of magnetic matter, and subject to further experiments on the permeability of magnetite, the material represented in Mr. Wilde's model by iron can not be basalt or magnetite, but must be iron.

If once it be granted that an iron shell may exist within the earth, there seems *a priori* to be no insuperable physical objection to the hypothesis that it may be of the thickness requisite to produce the secondary field of the earth, and yet at temperatures below that which would make it non-magnetic under ordinary pressures.

The low temperature of the ocean bottoms affords an adequate explanation of the difference of the magnetic properties of the hypothetical shell below the ocean and the continents respectively.

Finally, the whole investigation has been conducted on preliminary lines; not with the intention of attempting to decide any of the questions at issue, but with the view of determining whether they are worth discussing. The general impression left upon my own mind is, that though the fundamental hypotheses are beset with difficulty, they do not tend to any obvious absurdity, and that the relation between the permeability of the globe and its more pronounced geographical features is worth further attention.